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Corrigendum

Corrigendum to "Galois 2-extensions unramified outside 2" [J. Number Theory 124 (2007) 42–56]

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It has recently been kindly pointed out to the author by J.F. Jaulent that much of Theorems 2, 3 and 4 can be derived easily from [4, Theorem 3.5], which uses the concept of primitive ramification and descent and lifting theorem on p-rationality.

The notions of p-rational number fields or p-regular number fields were independently introduced by A. Movahhedi, T. Nguyen and Quang Do in [8] and by G. Gras and J.E. Jaulent in [2], and their relationships were studied and generalized in [4–7]. Their definition of p-rational number field is as follows. Let K be a number field. Let S denote the set of prime divisors of p in K. Then K is called p-rational if the Galois group G_S of the maximal pro-p extension of K unramified outside S is a free pro-p group.

However, the definition we have adapted includes the infinite primes of K in S. Observe that in their definition the infinite primes are not allowed to ramify, hence the infinite primes must split completely, unlike our definition where the infinite primes can ramify or split. Hence both the definitions coincide when K has no real embeddings. However, by our definition $\mathbb{Q}$ is not 2-rational, but by their definition $\mathbb{Q}$ is 2-rational. In [1], Gras introduces the concept of primitive ramification. Jaulent et al. [4, Theorem 3.5] give a characterization of Galois 2-extensions of $\mathbb{Q}$ which are 2-rational. However, for it to hold and to use $K = \mathbb{Q}$ as the base case, it is very important to use their definition of p-rationality.

To apply [4, Theorem 3.5] to our result, put $K = \mathbb{Q}$, $p = 2$ and L be a quadratic, biquadratic or degree 4 cyclic extension of $\mathbb{Q}$. $L/\mathbb{Q}$ is said to be primitively ramified if the set of places prime to 2 which are ramified in $L/\mathbb{Q}$ is 2-primitive for $\mathbb{Q}$. Primes $\equiv \pm 3 \bmod 8$ are 2-primitive primes of $\mathbb{Q}$. However, only one prime can be primitively ramified in $L/\mathbb{Q}$ as $\mathbb{Q}$ has a unique $\mathbb{Z}_2$-extension. Hence at most two finite primes, namely 2 and a prime $\equiv \pm 3 \bmod 8$, can ramify in

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$L/\mathbb{Q}$. Since $\mathbb{Q}$ is 2-rational (by their definition) and if L is imaginary, then L is 2-rational by their definition. However, for imaginary number fields both the definitions of 2-rationality coincide.

One of our main objectives in this paper is to give an explicit description of G_S using a result of Herfort, Ribes and Zalesskii [3,9] on virtually free pro-2 extensions, where S is the set of prime divisors of 2 and infinity, and this has not been touched upon by Jaulent and others.

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